Tree-structured multi-dimensional RARE for MIMO channel estimation

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High resolution MIMO (Multiple Input Multiple Output) channel estimation is considered. In the double-direction channel model the channel impulse response is described as a superposition of contributions from various discrete propagation paths, each of which characterized by its direction-of-departure at the Tx, direction of arrival at the Rx, the time delays of arrival and Doppler frequency. With exact knowledge of the sounding sequence the estimation problem reduces to a multi-dimensional harmonic retrieval problem. In this paper a tree-structured estimation scheme is proposed. Exploiting the rich multidimensional structure of the estimation problem allows to separate the unknown parameters. The harmonics along the various axis are efficiently estimated from the roots of the associated RARE- (RAnk Reduction Estimator) polynomials. The new algorithm enjoys simple implementation, high estimation accuracy, comparably mild uniqueness conditions and an automatic association of the parameter estimates. Simulation results using simulated and measured data recorded from a channel sounder illustrate the performance of the algorithm.

1 Introduction

Multi-dimensional (mD) harmonic retrieval problems arise in a large variety of important applications like signal separation problems in synthetic aperture radar, image motion estimation and chemistry applications. With the recently proposed double-directional channel model also the problem of MIMO communication channel estimation can be solved under this framework \cite{1}-\cite{5}. Here, the communication system is characterized by the direction-of-departure (DOD), direction of arrival (DOA), time delays of arrival (TDOA), the Doppler frequency, and the complex path weights of the dominant propagation paths. The individual channel transfer functions between all pairs of Tx and Rx antennas are measured using vector channel sounding. Applying mD Fourier transform we arrive at the conventional mD harmonic retrieval problem.

Numerous parametric and nonparametric estimation methods have been proposed for the one-dimensional (1D) exponential retrieval problem. Extensions of these techniques to the mD case are difficult to find and generally accompanied by a huge computational load \cite{6}. Simply applying 1D results separately in each dimension is not only a suboptimal solution in the sense that the benefits of the rich mD structure are not fully exploited, but also lead to poor estimation performance, difficulties in mutually associating the parameter estimates obtained in the various dimensions and over-strict uniqueness conditions \cite{7}. Contrariwise, many parametric high resolution methods specifically designed for mD frequency estimation require mD, highly nonlinear, and therefore non
convex optimization so that the computational cost associated with the optimization procedure becomes prohibitively high.

In this paper a novel eigenspace-based estimation method for mD-exponential retrieval problems is proposed. The method can be viewed as a mD extension to the RAnk Reduction Estimator (RARE) [8], originally developed for DOA estimation in partly calibrated arrays. We propose a tree structured estimation scheme that makes explicit use of the rich K-D structure of the measurement setup. The algorithm is computationally efficient due to its rooting-based implementation, yields automatically paired frequency parameter estimates, and shows strong estimation performance.

## 2 Signal Model

For simplicity we will start our considerations with the general mD sum-of-exponential model. For a detailed description on how this model relates to the double-directional channel model the interested reader is deferred to [1]-[3]. Consider the superposition of $L$ discrete time K-D exponentials corrupted by noise. The frequency parameters and the corresponding complex harmonics of the source signal along the various axis are denoted by $\omega_k^{(l)}$ and $z_k^{(l)} = \exp\{j \omega_k^{(l)}\}$ for $l = 1, \ldots, L$ and $k = 1, \ldots, K$, respectively. Here the superscript $(\cdot)^{(l)}$ is the source signal index and the subscript $(\cdot)_k$ specifies the frequency axis. Furthermore, let, the vector $h_k(\omega_k^{(l)}) = [1, \exp\{j \omega_k^{(l)}\}, \ldots, \exp\{j(M_k - 1) \omega_k^{(l)}\}]^T$ contain the contributions of the $l$th harmonic along the $k$th frequency axis with the sample support $M_k$. The Khatri-Rao (column-wise Kronecker) product of matrix $A$ and matrix $B$ is defined as, $A \circ B = [a_1 \otimes b_1, a_2 \otimes b_2, \ldots]$, where $a_k \otimes b_k$ is the Kronecker matrix product of $a_k$ and $b_k$. Introducing the parameter vector $\Omega = [\omega_1^T, \ldots, \omega_K^T]^T$, the K-D signal model associated with the harmonic retrieval problem can be formulated as

$$y(i) = H(\Omega)c(i) + n(i), \quad i = 1, \ldots, N$$ (1)

where the matrix of K-D complex exponentials

$$H(\Omega) \triangleq \begin{bmatrix} h(\omega_1^{(1)}, \omega_K^{(1)}), \ldots, h(\omega_1^{(L)}, \omega_K^{(L)}) \end{bmatrix} = H_1(\omega_1) \circ H_2(\omega_2) \circ \ldots \circ H_K(\omega_K)$$ (2)

also referred to as the steering matrix, is composed of $K$ individual matrices of 1D complex exponentials

$$H_k(\omega_k) \triangleq \begin{bmatrix} h_k(\omega_k^{(1)}), \ldots, h_k(\omega_k^{(L)}) \end{bmatrix} \in \mathbb{C}^{(M_k \times L)}$$ (3)

for $k = 1, \ldots, K$. The vector $y(i)$ contains the measurement data, $c(i)$ stands for the complex envelope of the $L$ harmonics, $n(i)$ is the vector of additive Gaussian noise and $N$ is the number of snapshots. Equation (1) describes the K-D harmonic retrieval problem which can efficiently be solved by ESPRIT type algorithms [6], [9] or more generally using (parallel factor) PARAFAC analysis ideas. In the following we derive a new search-free eigenspace-based estimation method for the general model in (1) which yields highly accurate estimates of the parameters of interest.

Let the data covariance matrix be given by

$$R \triangleq \mathbf{E} \{y(i)y^H(i)\} = E_S \Lambda_S E_S^H + E_N \Lambda_N E_N^H$$ (4)

where $(\cdot)^H$ denotes the Hermitian transpose, and $\mathbf{E} \{\cdot\}$ stands for statistical expectation. The diagonal matrices $\Lambda_S \in \mathbb{R}^{(L \times L)}$ and $\Lambda_N \in \mathbb{R}^{(m_1 - L) \times (m_1 - L)}$ contain the signal-subspace and the noise-subspace eigenvalues of $R$, respectively. In turn, the columns of the matrices $E_S \in \mathbb{C}^{(m_1 \times L)}$
and \( \mathbf{E}_N \in \mathbb{C}^{m_1 \times (m_1 - L)} \) denote the corresponding signal-subspace and noise-subspace eigenvectors for \( m_1 = \prod_{k=1}^{K} M_k \). The finite sample estimates are given by

\[
\hat{\mathbf{R}} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{y}(i)\mathbf{y}^H(i) = \hat{\mathbf{E}}_S \hat{\mathbf{A}}_S \hat{\mathbf{E}}_S^H + \hat{\mathbf{E}}_N \hat{\mathbf{A}}_N \hat{\mathbf{E}}_N^H.
\]

(5)

### 3 The K-D RARE Algorithm

The K-D root-MUSIC algorithm yields zero function values for the true parameter estimates, that is

\[
P_M(\omega_1, \ldots, \omega_K) = \mathbf{h}^T(1/z_1, \ldots, 1/z_K) \mathbf{E}_N \mathbf{E}_N^H \mathbf{h}(z_1, \ldots, z_K) = 0
\]

(6)

where \( z_k = e^{j\omega_k} \) denotes the exponential along the \( k \)-th axis. Equation (6) represents a polynomial equation in \( K \) variables which is generally hard to solve. The true source solutions can be obtained from the \( k \)-tuples \( \{(z_1^{(i)}, z_2^{(i)}, \ldots, z_K^{(i)})\}_{i=1}^{L} \) located on the unit circle that root the MUSIC criterion. The manifold vector in (6) can be represented as \( \mathbf{h}(z_1, z_2, \ldots, z_K) = \mathbf{T}_1(z_1) \mathbf{d}_2(z_2, \ldots, z_K) \) with \( \mathbf{d}_2(z_2, \ldots, z_K) = [\mathbf{h}_2(z_2) \otimes \ldots \otimes \mathbf{h}_K(z_K)] \in \mathbb{C}^{(m_2 \times 1)} \). Instead of solving the minimization problem (6) on the original manifold \( \mathcal{M} \triangleq \{ \mathbf{h}(z_1, \ldots, z_K) : |z_1| = \ldots = |z_K| = 1 \} \) we propose to relax the optimization problem searching for solutions on the so-called RARE manifold \( \mathcal{M} \triangleq \{ \mathbf{h}(z_1, c_1) : |z_1| = 1, \} \) for an arbitrary, finite-norm, and non-zero vector \( c_1 \in \mathbb{C}^{(m_2 \times 1)} \). Here, \( \mathbf{h}(z_1, c_1) = \mathbf{T}_1(z_1)c_1 \) with \( L_1(z_1) = [h_1(z_1) \otimes \mathbf{I}_{m_2}] \), \( m_2 = \prod_{k=2}^{K} M_k \) and \( \mathbf{I}_{m_2} \) denotes the \((m_2 \times m_2)\) identity matrix. The relaxed MUSIC criterion can now be formulated as

\[
P_R(z_1, c_1) = \mathbf{h}^T(1/z_1, c_1^*) \mathbf{E}_N \mathbf{E}_N^H \mathbf{h}(z_1, c_1) = c_1^T \mathbf{T}_1^T(1/z_1) \mathbf{E}_N \mathbf{E}_N^H \mathbf{T}_1(z_1) c_1 = c_1^T \mathbf{B}_1(z_1) c_1 = 0
\]

(7)

where \( \mathbf{B}_1(z_1) = \mathbf{T}_1^T(1/z_1) \mathbf{E}_N \mathbf{E}_N^H \mathbf{T}_1(z_1) \in \mathbb{C}^{(m_2 \times m_2)} \) denotes the RARE matrix polynomial. Interestingly, equation (7) has very simple solutions in the exponential \( z_1 \) which are given by the roots of the RARE polynomial

\[
P_R(z_1)||_{|z_1|=1} = \det\{\mathbf{B}_1(z_1)\}||_{|z_1|=1} = 0
\]

(8)

evaluated on the unit circle. It was proven in [8] that the solutions \( \{z_1^{(i)}\}_{i=1}^{L} \) to (8) and the solutions of the original MUSIC criterion (6) are identical provided that the condition

\[
L \leq m_2(M_1 - 1) = m_1 - m_2
\]

(9)

is satisfied. In other words the true parameter vector \( \omega_1 \) can uniquely be determined from the corresponding RARE polynomial without any knowledge of the remaining parameters \( \omega_2, \ldots, \omega_K \).

The computational cost required for evaluating the determinant of a matrix polynomial becomes prohibitively high if the dimension of the polynomial matrix is large [10]. In the following we derive an equivalent formulation for (8) based on a RARE matrix polynomial of reduced dimension.

Using the expansion rule for block matrix determinants equation (8) transforms to

\[
P_R(z_1)||_{|z_1|=1} = \det\{\mathbf{B}_1(z_1)\} = \det \{ \mathbf{T}_1^T(1/z_1) \mathbf{T}_1(z_1) - \mathbf{T}_1^T(1/z_1) \mathbf{E}_S \mathbf{E}_S^H \mathbf{T}_1(z_1) \} = 1/L \det \left[ \begin{array}{cc} \mathbf{I} & \mathbf{E}_S^H \mathbf{T}_1(z_1) \\ \mathbf{T}_1^T(1/z_1) \mathbf{E}_S & \mathbf{T}_1^T(1/z_1) \mathbf{T}_1(z_1) \end{array} \right] = m_1/L \det\{\mathbf{B}_1(z_1)\} = 0
\]

(10)
where \( \Delta_1 \triangleq T_1^T(1/z_1)T_1(z_1) \) is a constant diagonal matrix and \( B_1(z_1) \triangleq I - E_S^H T_1(z_1) \Delta_1^{-1} T_1^T(1/z_1) E_S \in \mathbb{C}^{(L \times L)} \) denotes the RARE matrix polynomial of dimension equal to \( L \), the number of discrete exponentials.

The estimation of the remaining parameter vectors \( \omega_2, \ldots, \omega_K \) is now straightforward. Due to the special structure of the steering matrix (2) the various dimensions of the parameter space can simply be interchanged. Corresponding RARE matrix polynomials in \( z_2, \ldots, z_K \) can be formulated following similar considerations as above and using an appropriate permutation of the columns of the signal eigenvectors in (4). The parameter vector \( \Omega \) is uniquely determined from the roots of these matrix polynomials under similar conditions as in (9).

A major obstacle emerging in this approach is the difficulty of correctly associating the frequency estimates, since the source parameters \( \omega_1, \ldots, \omega_K \) are separately obtained along the individual frequency axis. The computational cost associated with the pairing of the solutions is significant when the number of superposed signals is large and no efficient pairing procedure is available.

4 Tree Structured Dimensionality Reduction

Before introducing a tree structured estimation scheme that yields automatically associated frequency estimates we give a few notational conventions and point out some useful properties that are inherent in the structure of the estimation problem. Let the manifold vector

\[
d_k(z_k, \ldots, z_K) = \left[ h_k(z_k) \otimes \cdots \otimes h_K(z_K) \right] \in \mathbb{C}^{(m_k \times 1)} \quad \text{for} \quad m_k = \prod_{i=k}^{K} M_i
\]

contain only the frequency parameters along the \( k \)th to \( K \)th axis. The following recursive relation

\[
d_k(z_k, \ldots, z_K) = T_k(z_k)d_{k+1}(z_{k+1}, \ldots, z_K)
\]

hold for \( T_k(z_k) = (h_k(z_k) \otimes I_{m_{k+1}}) \). The manifold vector in (6) can then be represented as

\[
h(z_1, z_2, \ldots, z_K) = \bar{T}_{(k-1)}d_k(z_k, \ldots, z_K)
\]

with

\[
\bar{T}_k \triangleq \left[ T_1(z_1)T_2(z_2) \times \cdots \times T_k(z_k) \right].
\]

Provided that frequency parameters \( \{ \omega_i \}_{i=1}^k \) along the first \( k \) axis corresponding to one (or possibly multiple) specific source signal(s) are given, with \( z_i = \exp\{j\omega_i\} \) and making use of relation (13) we can rewrite the MUSIC criterion (6) as

\[
P_{M,k}(\omega_1, \ldots, \omega_K) = d_k^H(1/z_k, \ldots, 1/z_K) E_{N,k}^H E_{N,k}^T d_k(z_k, \ldots, z_K) = 0
\]

where \( E_{N,k}^H \triangleq E^H_{N} T_{(k-1)} \) and with \( z_k^* = 1/z_k \),

\[
\bar{T}_k^H = \left[ T_{k}^*(z_1)T_{k}^*(z_2) \times \cdots \times T_{k}^*(z_k) \right]^T = \left[ T_k^T(1/z_k) \times \cdots \times T_k^T(1/z_2)T_k^T(1/z_1) \right].
\]

Inserting (12) into (15) and following the considerations that led to (10) we obtain the polynomial equations

\[
P_{R,k}(z_k)|_{z_k=1} = m_k/L \det\{B_k(z_k)\}|_{z_k=1} = 0
\]
start, \( k := 1 \)

\[ \hat{E}_{S,1} = \hat{E}_S \]

evaluate matrix polynomial \( B_k(z_k) \) inserting \( \hat{E}_{S,k} \) into (18).

root RARE polynomial \( P_{R,k}(z_k) \) in (17).

select \( L_k \leq L \) “largest” roots \( \{z_k^{(l)}\}_{l=1}^{L_k} \) to obtain \( \{\hat{\omega}_k^{(l)}\}_{l=1}^{L_k} \)

and store root distance to unit circle (i.e. \( \{|z_k^{(l)}| - 1\}_{l=1}^{L_k} \)).

for \( \{z_k^{(l)} = e^{j\hat{\omega}_k^{(l)}}\}_{l=1}^{L_k} \) compute \( \{\hat{E}_{S,(k+1)}^{(l)}\}_{l=1}^{L_k} \) from (19).

store \( \hat{\omega}_k^{(1)} \).

set \( k := k + 1 \).

\[ \hat{E}_{S,k}^{(1)} \]

store \( \hat{\omega}_k^{(L_k)} \).

set \( k := k + 1 \).

if \( k = K + 1 \):

select \( L \) of the \( \prod_{i=1}^{K} L_i \) \( K \)-tuples that jointly minimize the Log-Likelihood function (21).\(^1\)

1) alternatively, select \( L \) \( K \)-tuples with minmax root distance to unit circle.

Figure 1: Tree structured \( K \)-D RARE.

for \( k = 1, \ldots, K \), where

\[
B_k(z_k) \triangleq I - \hat{E}_{S,k}^{H} T_k(z_k) \Delta_k^{-1} T_k^T (1/z_k) \hat{E}_{S,k} \in \mathbb{C}^{(L \times L)}
\]

(18)

\[
\hat{E}_{S,k} \triangleq T_k^{H} \hat{E}_S
\]

(19)

and \( \Delta_k \triangleq T_k^T (1/z_k) \Delta_{(k-1)} T_k(z_k) \) is a constant diagonal matrix. Given the frequency parameters \( \{\omega_i\}_{i=1}^{K} \) associated with a specific source the unknown frequency parameter \( \omega_k \) can uniquely be determined from the roots of (17) without any knowledge about the frequency parameters \( \{\omega_i\}_{i=k+1}^{K} \) along the the remaining axis. It can be shown that the uniqueness condition in (9) for \( k = 1 \) translates to

\[
L \leq m_{(k+1)}(M_k - 1) = m_k - m_{(k+1)}
\]

(20)

for \( k \in [1, \ldots, K] \) (see [8] for details). That is, provided that condition (20) is satisfied the roots of (15) and the roots of (17) located on the unit circle are exactly equivalent.

Due to finite sample and noise effects in the realistic case the \( L \) signal roots \( \{z_k^{(1)}, \ldots, z_k^{(L)}\} \) evaluated from (10) and (17) for \( k = 1, \ldots, K \), are displaced from its ideal positions on the unit circle. Note that all polynomials under consideration are self reciprocal, i.e. if \( z_k \) is a root then \( 1/z_k^* \) is also a root of the polynomial. In the following we consider only roots inside the unit circle. In order to avoid the difficulties associated with the pairing procedure we use the preceding results to develop a tree structured estimation scheme as indicated in Fig. 1. Each branch of the procedure tree represents a different candidate source, whereas the branch “depth” indicates the frequency axis of the frequency estimates. Following a specific branch at depth \( k \) the corresponding estimated signal eigenvector matrix \( \hat{E}_{S,k} \) is inserted into the RARE polynomial equation (17), the \( L_k \) largest roots inside the unit circle are determined and the corresponding root distance to the unit circle
is stored for the selection procedure in the final stage of the algorithm (for arbitrary choice of \( L_k \leq L \in \mathbb{N} \)). Then, for each of the \( L_k \) candidate roots the corresponding frequency parameters \( \{ \hat{\omega}_k^{(l)} \}_{l=1}^{L_k} \) are determined and the estimated signal eigenvector matrices are computed inserting \( \{ z_k^{(l)} = e^{j\hat{\omega}_k^{(l)}} \}_{l=1}^{L_k} \) into (19). At this point each estimate \( \{ \hat{\omega}_k^{(l)} \}_{l=1}^{L_k} \) together with its corresponding signal eigenvector matrices \( \{ \hat{E}_S^{(l)}(k+1) \}_{l=1}^{L_k} \) branches into \( L_k \), incrementing the branch depth index \( k \) by one. This procedure is continued until all branches arrive at the branch depth \( k = K + 1 \). The frequency estimates obtained along the individual branches from branch depth one to branch depth \( K \) form the candidate \( K \)-tuples of frequency estimates. In the final step of the estimation scheme \( L K \)-tuples are selected according to an appropriate selection criterion. Various selection criteria are possible. A simple selection procedure is to choose the \( K \)-tuples according to the root distances of the estimates obtained along the individual branches using minimum average root distance or minmax root distance criteria. A more reliable but also more complex selection scheme is to pick the \( L K \)-tuples which minimize the conditional Log-Likelihood function

\[
L_c(\Omega) = \text{trace}\{ \mathbf{H}(\Omega) (\mathbf{H}^H(\Omega)\mathbf{H}(\Omega))^{-1} \mathbf{H}^H(\Omega) \hat{\mathbf{R}} \}.
\]

(21)

5 Simulation Results

5.1 Synthetic Data

In this section simulation results using synthetic data are presented. Computer simulations are performed for the 3D case with sample sizes \( M_1 = M_2 = M_3 = 5 \). The \((5^3 \times 5^3)\) data covariance matrix is computed from \( N = 5^3 \) snapshots and a number of \( L = 3 \) equi-powered exponentials is assumed with the frequency parameters \( \omega_1 = (0.3193\pi, 0.3231\pi, 0.6193\pi) \), \( \omega_2 = (0.4063\pi, 0.4151\pi, 0.6771\pi) \) and \( \omega_3 = (0.2343\pi, 0.5234\pi, 0.7771\pi) \). The RMSE of the parameter estimates \( \hat{\omega}_1, \hat{\omega}_2 \) and \( \hat{\omega}_3 \) obtained by the \( K \)-D RARE algorithm averaged over 100 simulation runs are plotted versus the SNR in Fig. 2. A comparison to the individual CRBs reveals that the new method yields estimation performance close to the optimal bound.
5.2 Measurement Data

The measurement data used for this paper was recorded during a measurement campaign in Weikendorf, a suburban area in a small town north of Vienna in autumn 2001. Measurements were performed by a vector channel sounder RUSK ATM, manufactured by MEDAV [1]. The sounder operated at a center frequency of 2 GHz with an output power of 2 Watt and a transmitted signal bandwidth of 120 MHz. At the mobile station a uniform circular array composed of 15 monopoles arranged at an inter-element spacing of 0.43λ was mounted on top of a small trolley at a height of about 1.5m. At the receiver site a uniform linear array\(^1\) composed of 8 elements with half wavelength distance between adjacent sensors was mounted on a lift in about 20m height. With above arrangement, consecutive sets of the (15 × 8) individual transfer functions, cross-multiplexed in time, were measured from correlation analysis of the received and the transmitted signal. The acquisition period of one snapshot was limited to 3.2µs corresponding to a maximum path length of about 1km. A constant number of 11 discrete propagation pathes was assumed. A full rank sample covariance matrix with sample support \(M_1 = 15\) (TDOA axis) and \(M_2 = M_3 = 5\) (DOA/DOD axis) was formed using spatial smoothing techniques. During the measurements the receiver was moved at speeds of about 5km/h on the sidewalk. Rx-position and Tx-position, as well as the motion of the transmitter are marked in the site map on the left side of Fig. 3. The transmitter was passing through a pedestrian tunnel approximately between times \(t = 25\) s and \(t = 30\) s of the measurement run. TDOA, DOA and DOD estimates obtained with 3D RARE are displayed on the right side of Fig. 3 relative to the orientation of the arrays. For TDOA and DOA estimation the polynomial rooting based technique was implemented as described in the previous section while for DOD estimation a 1D spectral search was performed instead of rooting. In the estimation procedure a limited field-of-view of about 60 and 120 was used for DOA and DOD angle estimation, respectively. The results show that during the first 25 seconds the propagation scenario is dominated by a strong line-of-sight component surrounded by local scattering pathes from trees and buildings. The trace of the DOA estimates in Fig. 3 and also the corresponding TDOA and DOD estimates match the motion of the transmitter for the direct path. At time 25 the trolley reaches the pedestrian tunnel and a second path resulting from scattering at the building (see Fig. 3) appears at a DOA of approximately \(-15^\circ\). This path corresponds to a significantly larger access delay of about 0.425 to 0.625 µs. By the time the Tx moves out of the tunnel the dominant LOS component with local scattering is newly tracked by the 3D-RARE algorithm.

\(^1\)provided by T-NOVA, Germany
6 Summary and Conclusions

A novel method for $K$-D harmonic exponential estimation has been derived as a mD extension of the conventional RARE algorithm. High resolution frequency parameter estimates are obtained from the proposed method in a search-free procedure at low computational complexity. Based on the rich structure of the $K$-D measurement model a tree-structured estimation procedure has been introduced that yields automatically associated estimates of the parameters of interest. Simulation results based on synthetic and measured data of a MIMO communication channel underline the strong performance of the new approach.

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